The fascinating realm of quantum mechanics and its implications

Felipe Taha Sant'Ana

Institute of Physics, Polish Academy of Sciences

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INSTYTUT FIZYKI POLSKIEJ AKADEMII NAUK

NATIONAL SCIENCE CENTRE POLAND

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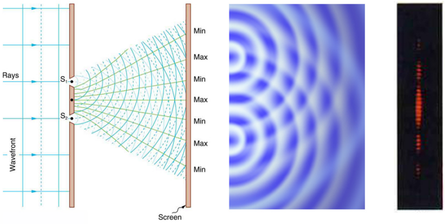
Summary

Let there be light!

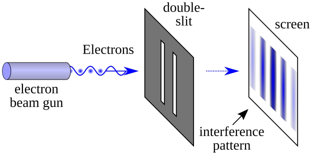
1801: Thomas Young double slit experiment

- First experimental demonstration of wave nature of light
- IT IS A WAVE!



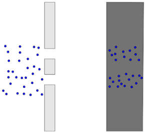
(a) (b) (c)

- 1927: Davisson and Germer experiment



Instead, if light behaved as individual particles

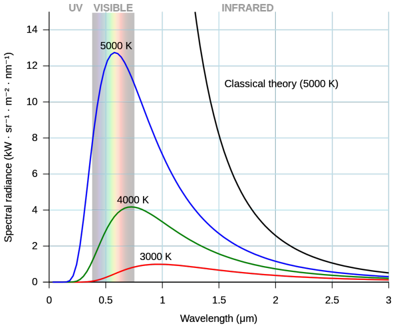
- Two slits would produce two distinct clusters of particles.



What's up classical mechanics??

1900: Max Planck's solution to the black body radiation

- What is a black body? An idealized opaque, non-reflective body in thermal equilibrium that emits radiation in a specific, continuous spectrum of wavelengths, that depends only on the body's temperature
- Classical prediction: Rayleigh-Jeans law $u(\nu) \propto \nu^2$ $\rightarrow$ Ultraviolet catastrophe



- Planck's proposal: Energy is quantized $E = h\nu$, *quanta* $\rightarrow$ later *photon*
- Together with Boltzmann statistic theory, distribution $p \sim e^{-E/k_B T}$, Planck derived what is known as Planck's law for the energy density of a black body: $u(\nu, T) \propto \frac{\nu^3}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$.

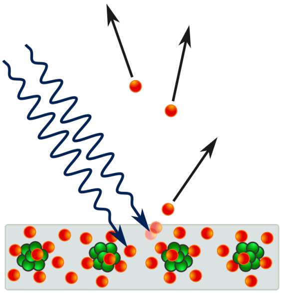
Here comes Einstein

- Photoelectric effect is the emission of electrons from a material caused by electromagnetic radiation, such as light;
- In 1902, Philipp Lenard observed that the energy of individual emitted electrons was independent of the applied light intensity. This appeared to be at odds with Maxwell's wave theory of light, which predicted that the electron energy would be proportional to the intensity of the radiation.

1905: Albert Einstein's photoelectric effect

- Einstein's proposal: following Planck's quantization hypothesis, the energy of the photoelectrons depend on the frequency of the incident light,

$$E_{electrons} = h\nu - \Phi.$$

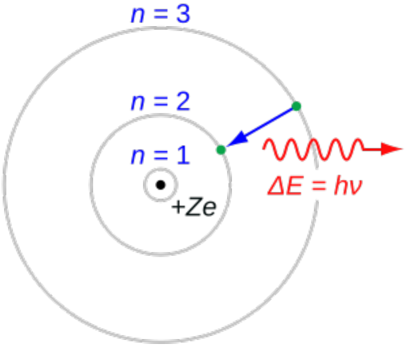


The atom is born!

1913: Niels Bohr atomic theory

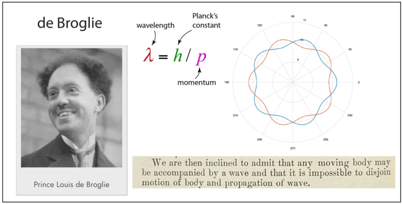
- Built upon Rutherford's nuclear model: positive core with electrons orbiting it
- Electrons orbit the nucleus in fixed and quantized energy levels
- Energy levels of an electron in a hydrogen atom:

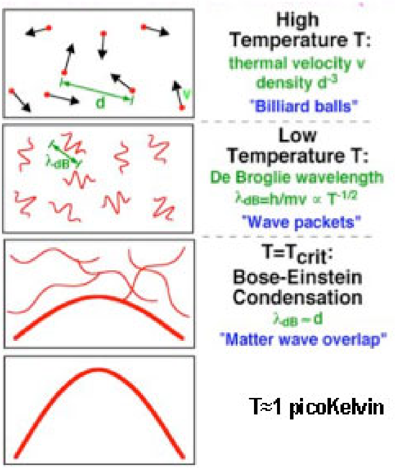
$$E_n = -\frac{m_e e^4}{8\epsilon_0^2 h^2 n^2} = -\frac{13.6eV}{n^2}$$



1924: de Broglie and his wavelength

- The French physicist Louis de Broglie, in 1924, proposed that every moving particle with momentum p can be associated with a wavelength λ:





- Thermal wavelength: $\lambda \propto 1/\sqrt{T}$
  - Bose-Einstein condensation = Very active research topics: cold atoms, cooling techniques, superfluids, quantum vortices, quantum phase transition, etc...

How much mechanics

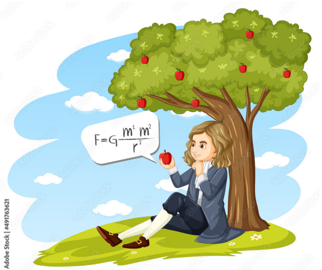
- The term *quantum mechanics* was coined by a group of physicists including Max Born, Werner Heisenberg, and Wolfgang Pauli, at the University of Göttingen in the early 1920s
- It was first used in Born's 1925 paper "Zur Quantenmechanik"
- The word *quantum* comes from the Latin word for "how much"

The infamous Schrödinger equation

- In 1925, the German physicist Erwin Schrödinger derived an equation for the time evolution of a quantum mechanical system, described by the Hamiltonian H.

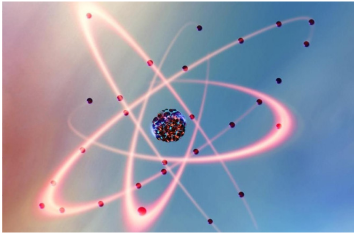
Classical Mechanics: Newton

$$F = \frac{dp}{dt}$$



Quantum Mechanics: Schrödinger

$$i\hbar \frac{d}{dt}|\Psi\rangle = H|\Psi\rangle$$



- Example: $i\hbar\partial_t \Psi(x,t) = \left(-\frac{\hbar^2}{2m}\partial_x^2 + V(x,t)\right)\Psi(x,t)$

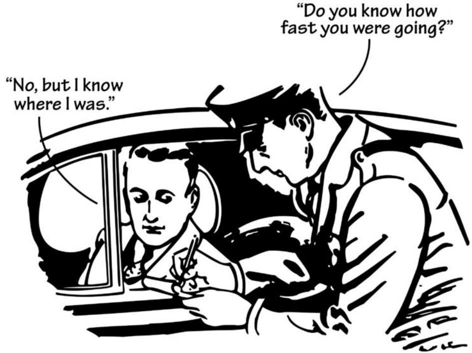
1926: Max Born's rule

- Born's Rule is a fundamental principle in quantum mechanics that provides the link between the mathematical formalism of the theory and experimental observations, specifically the probabilities of measurement outcomes. Proposed by physicist Max Born in 1926, it describes how to calculate the probability of finding a quantum system in a particular state after a measurement.
- Example: Consider a quantum system described by the wave function $\psi(x)$. Then we can assign a probability density for finding the particle at position x by $|\psi(x)|^2$.
- It implies that if you sum over all possible position, you have 100% chance of finding the particle, i.e. $\int dx\, |\psi(x)|^2 = 1$.

Where am I? The uncertainty principle

- 1927: German physicist, Werner Heisenberg states that there is a limit to the precision with which certain pairs of physical properties, such as position and momentum, can be simultaneously known. In other words, the more accurately one property is measured, the less accurately the other property can be known:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$



Heisenberg takes full responsibility

Vacuum is not empty after all: quantum fluctuations are a consequence of $\Delta E \Delta t \geq \hbar/2$. Pairs of virtual particles are continuously created and annihilated due to random fluctuations in values for the fields. Two notable consequences:

1. **Hawking radiation**

Close to the event horizon, a virtual particle pair forms out of quantum fluctuations, and instead of annihilating each other, one particle falls into the black hole while the other escapes. The escaping particle is known as Hawking radiation.

2. **Casimir effect**

Two uncharged, conducting plates placed in a vacuum experience an attractive force due to quantum fluctuations of the electromagnetic field. This effect was first predicted by Dutch physicist Hendrik Casimir in 1948.

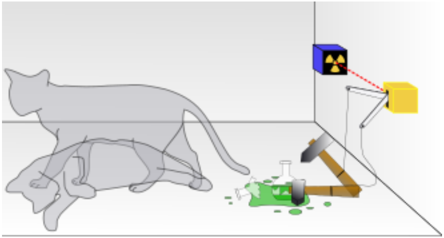
Where is my cat, Mr. Schrödinger?

The 1935 discussion with Einstein

Thought experiment: before looking inside the box, the cat is both dead and alive!

The Copenhagen interpretation

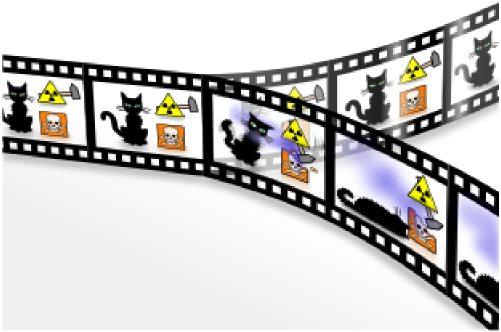
After performing a measurement, the system is in one state of the superposition.



$$|\Psi\rangle_{CAT} = \frac{|DEAD\rangle \pm |ALIVE\rangle}{\sqrt{2}}$$

The many worlds interpretation

- It was first proposed in 1957 by American physicist Hugh Everett;
- In this interpretation, every quantum event is a branch point; the cat is both alive and dead, even before the box is opened, but the "alive" and "dead" cats are in different branches of the multiverse, both of which are equally real, but which do not interact with each other;
- There is no wave function collapse, since every possible outcome of a quantum event exists in its own universe.



The Einstein attack

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

The EPR paper

- Einstein-Podolsky-Rosen paradox: a thought experiment showing that the quantum mechanical description of reality provided by wave functions is not complete

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Entangled states

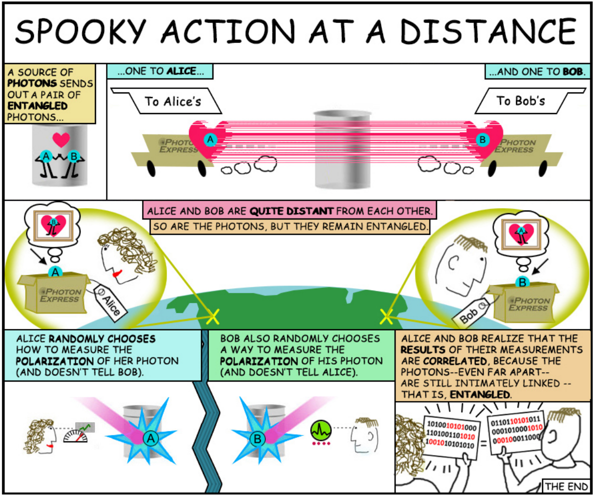
Protocol: Consider two entangled particles, A and B. The system is prepared in the following entangled state

$$|\Psi\rangle = \frac{|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B}{\sqrt{2}}$$

Now let Alice be the person who measures the state of particle A, and Bob the one who measures the state of particle B. Possibilities:

- If Alice measures $|0\rangle_A$ for particle A, the state will collapse and it will be in the state $|0\rangle_A|1\rangle_B$. This means that Bob will measure $|1\rangle_B$, with 100% chance, the state of particle B;
- If Alice measures $|1\rangle_A$ for particle A, the state will collapse and it will be in the state $|1\rangle_A|0\rangle_B$. This means that Bob will measure $|0\rangle_B$, with 100% chance, the state of particle B.

"Spukhafte Fernwirkung, mate"



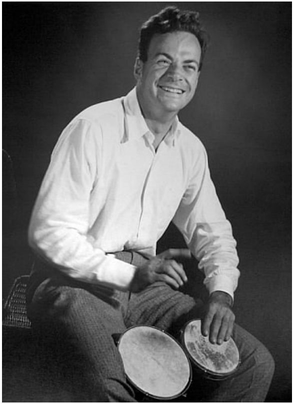
Counter-intuitive means wrong, right? Not so fast, speedy!

Experimental confirmations

1. Double-slit: Demonstrated that particles, such as electrons, can show wave-like behaviors;
2. Stern-Gerlach: In 1922 it was demonstrated that particles have intrinsic angular momentum, or spin, that comes in certain discrete values proportional to the Planck's constant;
3. Bell's inequality test: in the 1980s, a series of experiments confirmed the phenomenon of quantum entanglement;
4. Quantum Electrodynamics (QED): theory that describes how light and matter interact, has been confirmed by measuring the electron's anomalous magnetic moment to an accuracy of 10^{-9} compared to QED predictions;
5. The lamb shift, i.e. anomalous difference in the energy between two electrons in the $^2S_{1/2}$ and $^2P_{1/2}$ orbitals of the hydrogen atom, and the fine structure constant, which quantifies the strength of the electromagnetic interacting between charged particles, have been measured in experiments with high accuracy compared to quantum mechanical prediction.

Surely you're joking, Mr. Feynman!

"If you understand quantum mechanics, you do not understand quantum mechanics"



So, what is quantum mechanics after all?

1. It is a probabilistic theory to describe the subatomic world. Contrary to deterministic theories, where you can calculate an outcome through a set of variables, in quantum mechanics all you can hope is to acquire a combination of probable outcomes for the dynamics of the system.

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3. Different interpretations: When you make a measurement over the system, you collapse its wave function, resulting in the system acquiring one of all possible states. There are other interpretations as well, such the many worlds, where all possible outcomes do realize, but in different universes that you do not have access to. Philosophical implications of quantum mechanics is an active field of research.

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5. How can we use it to our advantage?

Quantum Computation

Qubit power

- 1 Bit $=$ 0 or 1;

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Example 300 qubits = 2^{300} states $\sim 10^{90}$ is larger than the number of atoms in the universe $\sim 10^{80}$

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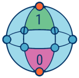
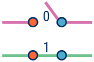
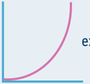
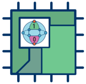
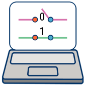
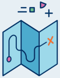
- Number factorization:
  1. Classical computer: General Number Field Sieve (GNFS) $t = \mathcal{O}\left(\exp\left((64/9)^{1/3}(\log n)^{1/3}(\log\log n)^{2/3}\right)\right)$ = thousands of years for a 300-digit number
  2. Shor's algorithm: $t = \mathcal{O}\left((\log n)^3\right)$ = minutes for a 300-digit number

Quantum Computing		Vs.	Classical Computing	
	Calculates with qubits, which can represent 0 and 1 at the same time		Calculates with transistors, which can represent either 0 or 1	
	Power increases exponentially in proportion to the number of qubits		Power increases in a 1:1 relationship with the number of transistors	
	Quantum computers have high error rates and need to be kept ultracold		Classical computers have low error rates and can operate at room temp	
	Well suited for tasks like optimization problems, data analysis, and simulations		Most everyday processing is best handled by classical computers	

CBINSIGHTS

Not yet

1. Error correction: Quantum computers are extremely sensitive to noise and errors caused by interactions with the environment. Therefore, error correction techniques is essential for building practical quantum computers;
2. Scalability: Scaling up quantum computers to thousands of qubits while maintaining high levels of coherence and low error rates remains a major challenge;
3. Hardware: Developing high-quality quantum hardware remains very challenging, such as qubits and control electronics;
4. Software: Quantum algorithms and software development tools are still in the early days, so in order for quantum computing to reach its full potential, much progress has yet to happen in this direction;
5. Classical computers interface: Quantum computer will not replace classical ones; they will serve as complementary technology. Developing efficient and reliable methods for transferring data between them is essential for practical applications;
6. Very expensive.

Acknowledgment

Thank you!

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