

always disagree



same tallies



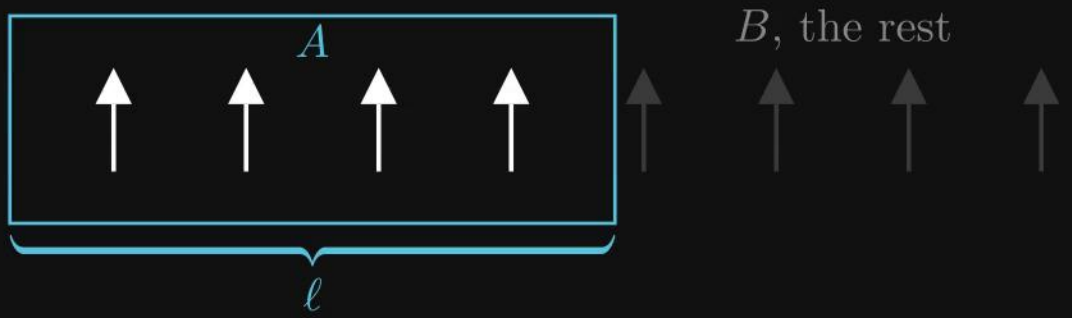
$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

? Same tallies. Is there any difference at all?

$$\ell = |A|$$

today: a few spins

by Part 4: a long window in an endless chain

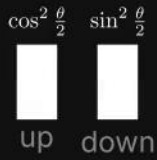


$$|\psi\rangle = \cos\frac{\theta}{2}|\uparrow\rangle + \sin\frac{\theta}{2}|\downarrow\rangle$$

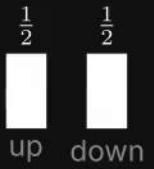


top view: x-y, home of every rotation about z

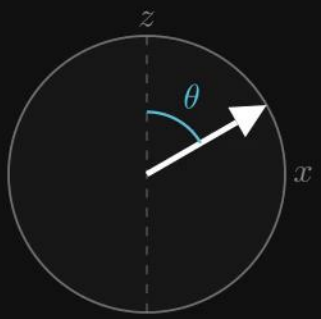
side view: x-z



same odds, different objects



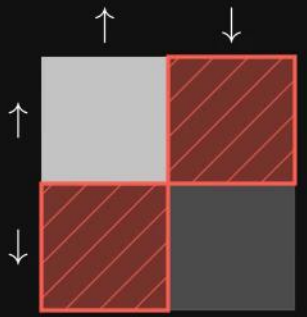
tilt θ



side view: x-z



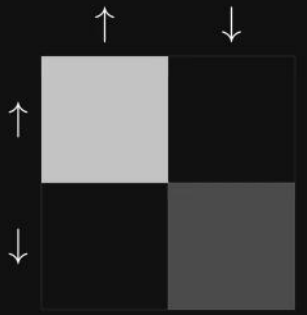
colour = size of the entry, $|\rho_{ij}|$, black 0 to white 1



$$\rho = \begin{pmatrix} \cos^2\frac{\theta}{2} & \cos\frac{\theta}{2}\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2}\sin\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix}$$

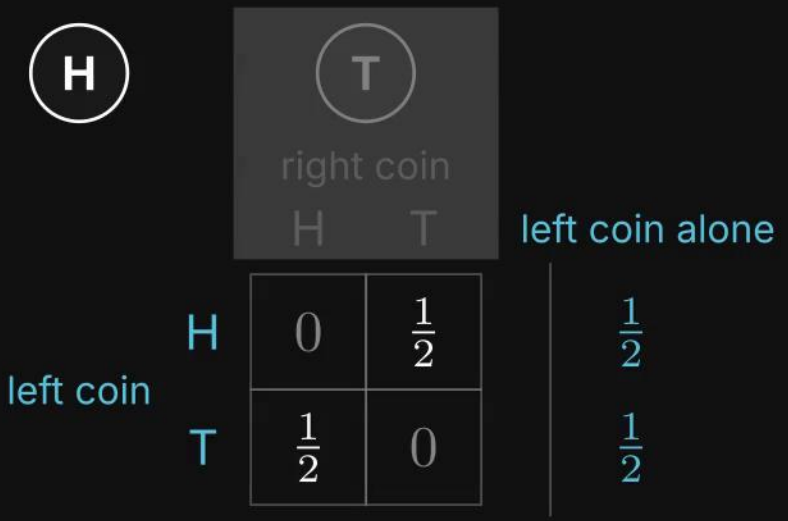
red corners:
what a coin never has

density matrix ρ



a coin loaded to the same odds

$$\begin{pmatrix} \cos^2\frac{\theta}{2} & 0 \\ 0 & \sin^2\frac{\theta}{2} \end{pmatrix}$$



$$p_A(i) = \sum_j p(i,j)$$

add



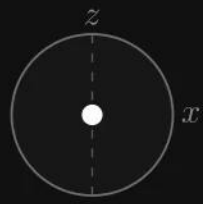
$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

BLACK BOX · `partial trace`

- The same rule applied in any other basis gives the same reduced density matrix (basis independence; asserted, not shown).
- It predicts every measurement made on A alone, in any direction.
- A cell is kept and added, or discarded; nothing else.

A alone

	↑	↓
↑	$\frac{1}{2}$	0
↓	0	$\frac{1}{2}$

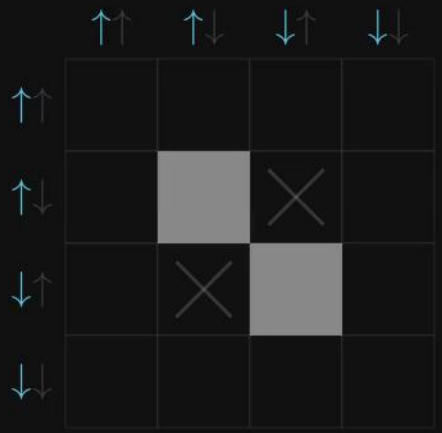


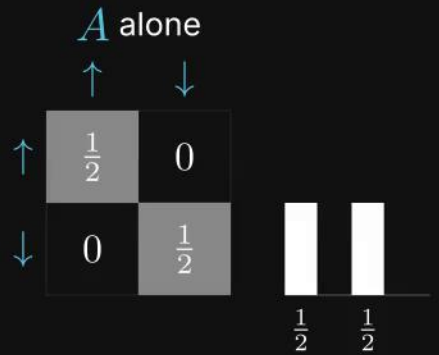
side view: x-z

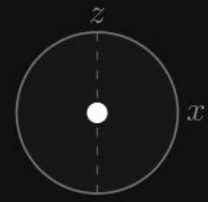
$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi| = \tfrac{1}{2}\mathbb{1}$$



$$|\Psi\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle - \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle$$







eigenvalue bars of ρ_A

$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi| = \tfrac{1}{2}\mathbb{1}$$

Part 3 keeps these bars when it compresses a chain

? PAUSE AND PONDER

Is the left spin of the singlet in a definite state?

Vote: Yes, one we simply have not found | No, and no measurement on A alone can tell it from a fair coin | It depends on what B does



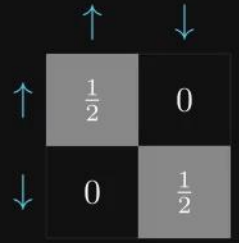
$$|\Psi\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle - \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle$$

uncertainty (ruler built next scene)

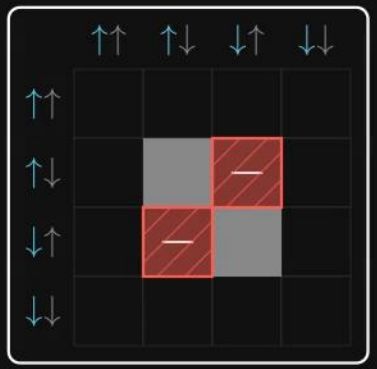
	whole	part
coins	$\log 2$	$\log 2$
singlet	0	$\log 2$

Only quantum lets the part know less than the whole.

$$S(\rho_A) = -\sum_a p_a \log p_a$$



one spin of the singlet



the whole singlet



a pure state:
one eigenvalue equal to one

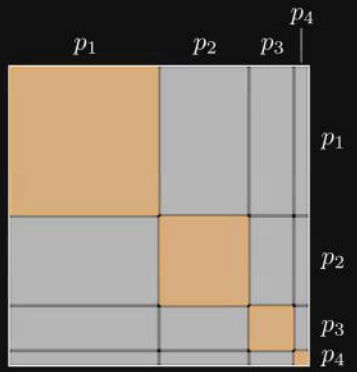
	uncertainty	
	whole	part
coins	$\log 2$	$\log 2$
singlet	0	$\log 2$

$n = 2$ is the entropy you read by overlapping two copies; Part 2 needs exactly that trick.



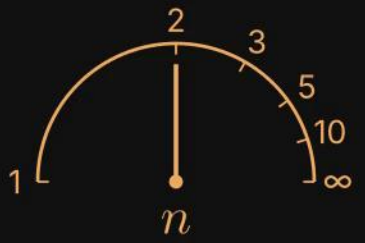
eigenvalue bars p_a





overlap area

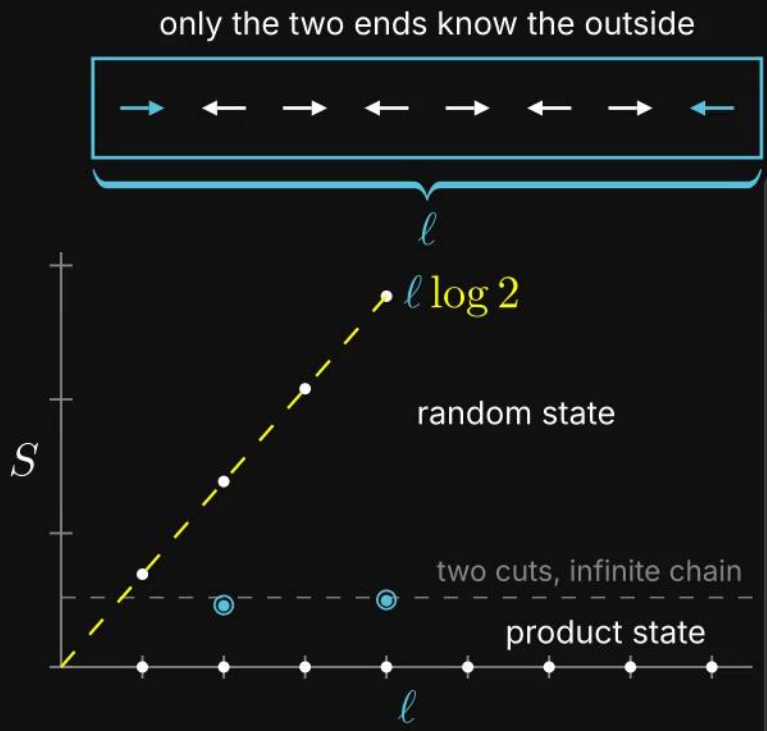
$$\sum_a p_a^2 = \mathrm{Tr}\rho^2$$



$$S^{(n)} = \frac{1}{1-n} \log \mathrm{Tr}\rho^n$$

$$\mathrm{Tr}\rho^n = \sum_a p_a^n$$

$n = 2$: minus the log of an overlap, $S^{(2)} = -\log \mathrm{Tr}\rho^2$



ground state of a small magnet

- a small ring
- a longer ring

$S(\ell) \to \text{const}$ (gapped)

BLACK BOX · area law

- Hastings 2007: gapped ground state of a chain: S(ℓ) stays below a constant for every ℓ; the proof uses only the gap and locality
- A gap makes correlations decay exponentially (Hastings-Koma 2006); back in Part 3
- Gapless chains grow like log ℓ instead; not our case

TAKE-HOME

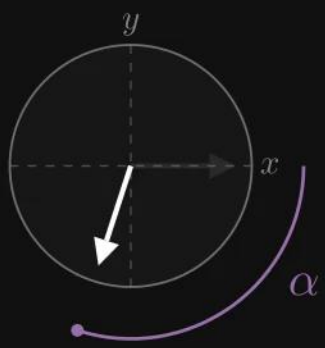
$$S = -\sum_a p_a \log p_a$$

- A density matrix is a table of every prediction: the diagonal is odds, the red corners are what a coin never has.
- The curtain (partial trace): keep the cells whose B labels agree, add them, discard the rest; the singlet's red corners are discarded and the part becomes a fair coin.
- The Meter runs on eigenvalue bars, not diagonal entries; for a gapped ground state it stops growing with the window.

CAVEATS

- ☐ The partial trace gives the same reduced density matrix in every basis (asserted in 1.5b).
- ☐ 0 log 0 = 0; eigenvalues of a density matrix are ≥ 0 and sum to 1; $S \geq 0$ with $S = 0$ only for a pure state (stated, not proved).
- ☐ The $n \to 1$ limit of $S^{(n)}$ was named, not derived (1.8).
- ☐ Area law (Hastings 2007) and gap ⇒ exponential decay of correlations (Hastings–Koma 2006): taken on faith (1.9).
- ☐ Random state: $\ell \log 2$ only for $\ell \ll L/2$; the ring points are toys at even $\ell \leq L/4$ on two small rings (1.9).
- ☐ The two Bloch views stand in for the sphere (1.3).
- ☐ The rotation about z by alpha was shown, not defined; its two letters arrive in 2.2 (1.10).

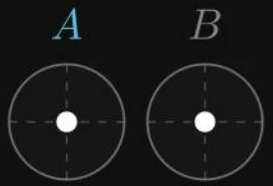
the tilted spin



top view: x-y

changed

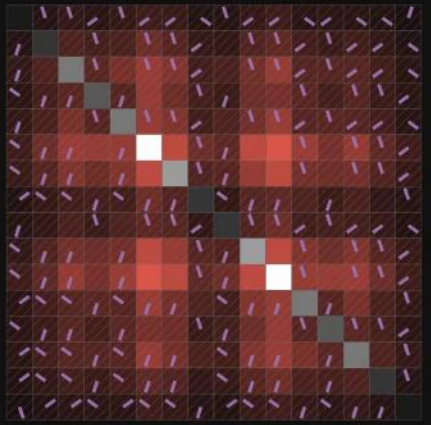
the singlet



$$|\Psi\rangle = \tfrac{1}{\sqrt{2}}\big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\big)$$

unchanged

the window's table, ρ_A



colours unchanged, needles spinning



the Meter is blind

? Which broke a rule?

The Meter cannot even tell that the red corners exist; a rotation spins their needles, and averaging over all rotations will wipe them out.

Part 2 builds a meter for the corners and finds it needs no entanglement.