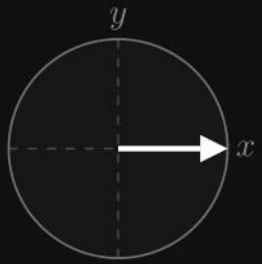
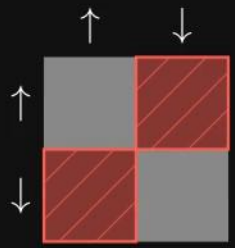
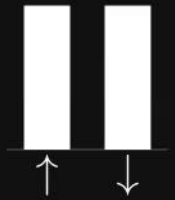
pure table against coin:
here the eigenvalue bars differ too, and that the Meter can see

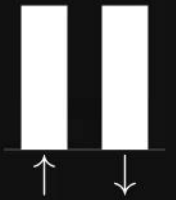


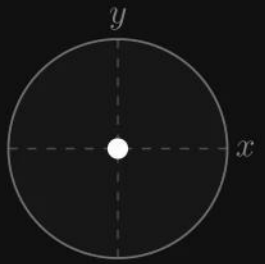
$|+x\rangle$



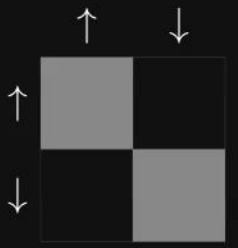


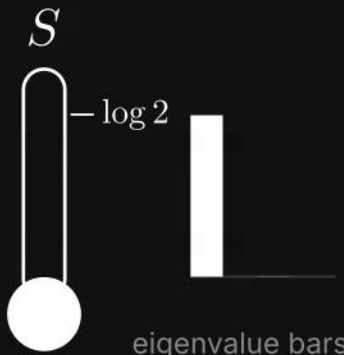
same histogram





fair coin

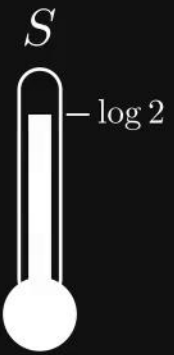




eigenvalue bars

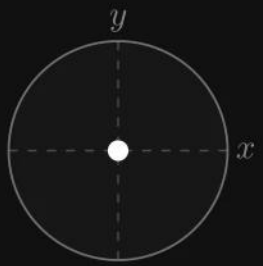


eigenvalue bars

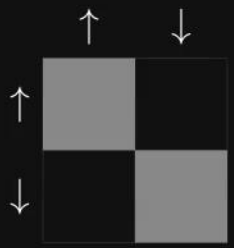


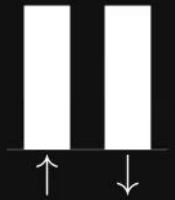
$$\rho_{+x} = \tfrac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \rho_{\text{mix}} = \tfrac{1}{2}\mathbb{1}$$

a friend measures along z and does not tell

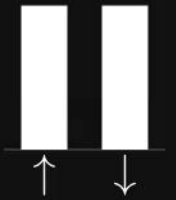


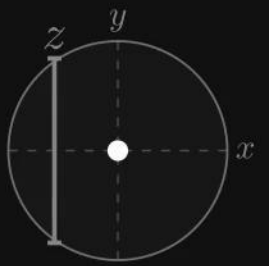
$|+x\rangle$



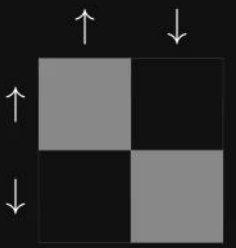


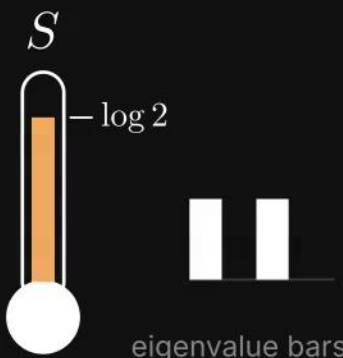
same histogram





fair coin



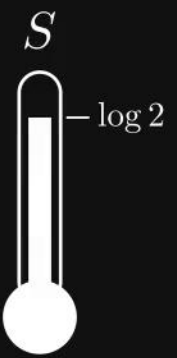


eigenvalue bars

the secret cost $\log 2$



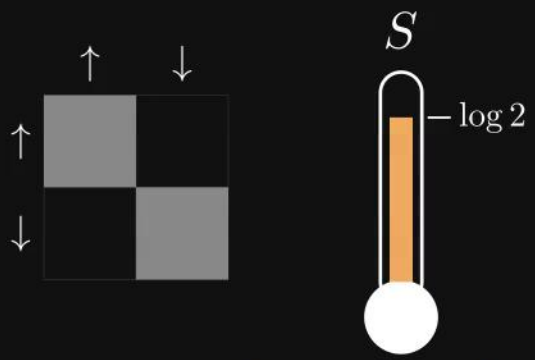
eigenvalue bars



cost 0

$$\rho_{+x} = \tfrac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \rho_{\text{mix}} = \tfrac{1}{2}\mathbb{1}$$

2023: the asymmetry is defined (Ares, Murciano and Calabrese).
2024: microscopic origin of the quantum Mpemba effect (Rylands and co-workers); the effect seen with trapped ions (Joshi and co-workers).
2025: a review of the quantum Mpemba effects (Ares, Calabrese and Murciano).
Today: the same meter pointed at a gapped magnet, with nothing fitted.

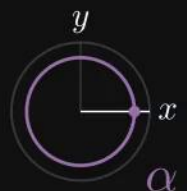


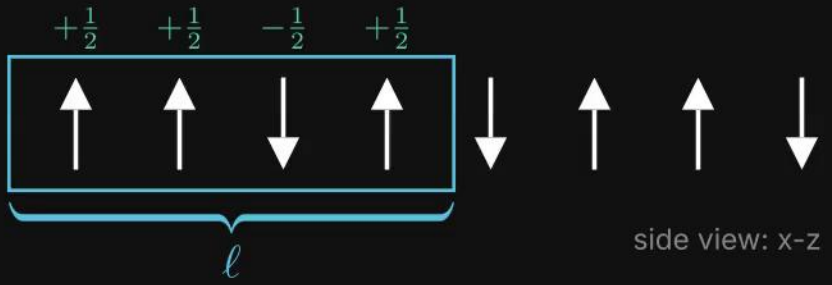
the secret cost $\log 2$

? What did the secret cost?

BLACK BOX · the 2023 definition

- The definition is taken from Ares, Murciano and Calabrese (2023) and the 2025 review; this series rebuilds its picture, not its history.
- Quantum Mpemba effect (Rylands and co-workers 2024; trapped ions, Joshi and co-workers 2024): a state that starts further from the symmetry can restore it faster after a sudden change; named here, never derived.
- The paper's own use of the meter: at equilibrium it reads how far a window sits from a symmetry that the ground state breaks.



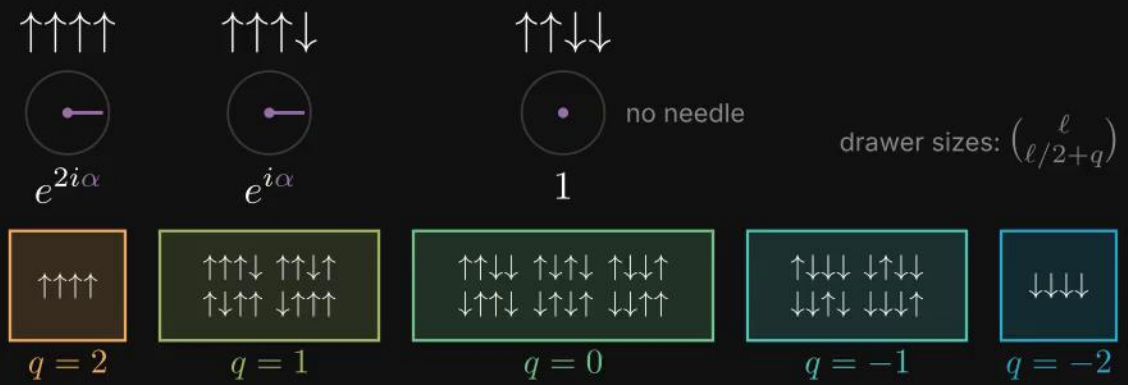


side view: x-z

ups minus downs, halved:

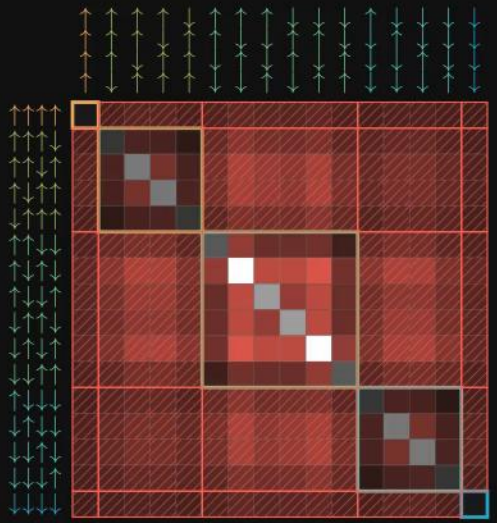
$$Q_A = 1$$

$$Q_A = \frac{1}{2}\sum_{j\in A}\sigma_j^z$$



$$e^{i\alpha Q_A}|q\rangle = e^{i\alpha q}|q\rangle$$

odd windows: half-integer q, integer differences



window ℓ of a small magnet

(a ring of L spins, couplings Δ_z, γ)

$q = 2$
$q = 1$
$q = 0$
$q = -1$
$q = -2$

weight of a block = the sum of the squared sizes of its cells

one apart:

two apart:

three apart:

same drawer:

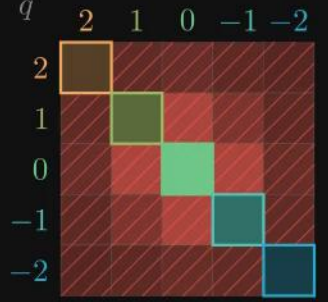
keep every block; another state of this magnet (Part 3) leaves the odd distances dark

$$\rho_A = \sum_{q,q'} \Pi_q \rho_A \Pi_{q'}$$

$q = q'$: the five diagonal blocks

$q \neq q'$: the red blocks

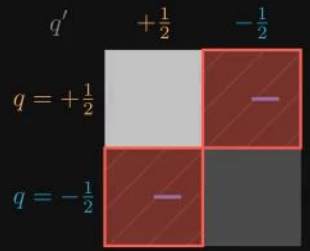
Π_q keeps what sits in drawer q



brightness: square root of the weight

one tilted spin

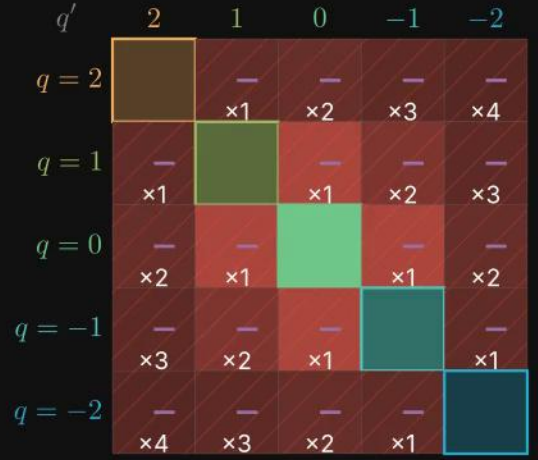




one spin: a difference of 1

colour fixed, needle turns

the block cartoon of the window ℓ

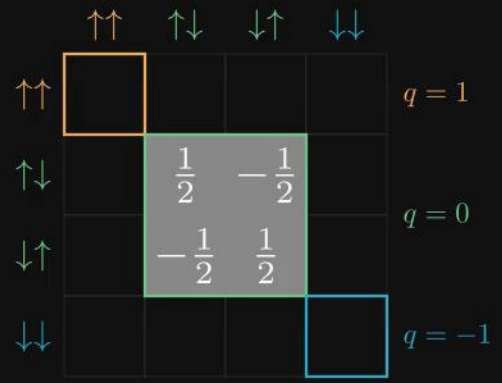


brightness: square root of the weight

$$\rho_{qq'} \mapsto e^{i\alpha(q-q')}\rho_{qq'}$$

speed = charge difference

the Bell pair, sorted by charge



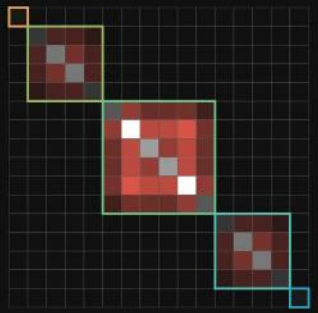
same charge, no needle to turn

PAUSE AND PONDER

A friend measures the window's charge and never tells you the result. Is the window now more certain, less certain, or the same?

Vote: More certain | Less certain | The same

a friend measures Q_A and does not tell





$q = 2$
$q = 1$
$q = 0$
$q = -1$
$q = -2$

$$\rho_{A,Q} = \sum_q \Pi_q \rho_A \Pi_q$$

$\rho_{A,Q}$: the table after the secret measurement

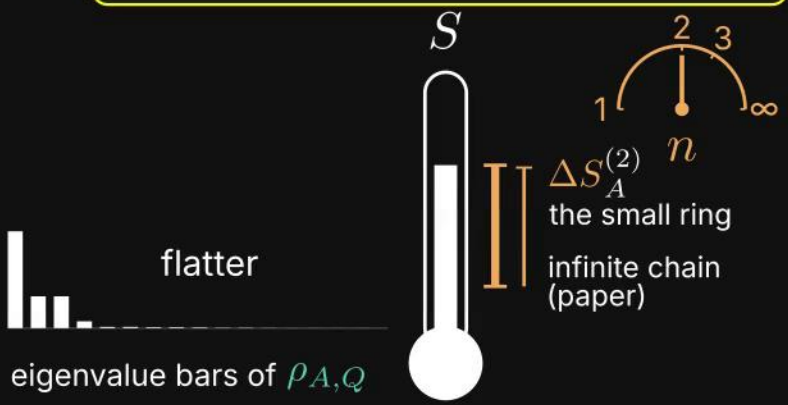
BLACK BOX · ΔS ≥ 0

- Erasing the off-block cells never lowers the Meter: ΔS ≥ 0 for every n, with equality exactly when there is nothing to erase ([ρA, QA] = 0).
- Reason, on the Caveats: the erased table is an average of rotated copies (2.8), and averaging tables never sharpens eigenvalue bars.
- The asymmetry is a difference of two Meter readings on two tables, so it can be log 2 while the Meter itself reads 0 (2.6).

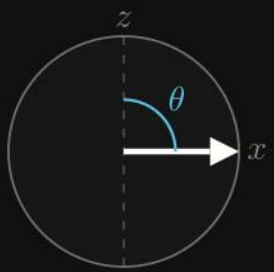
? PAUSE AND PONDER

A friend measures the window's charge and never tells you the result. Is the window now more certain, less certain, or the same?

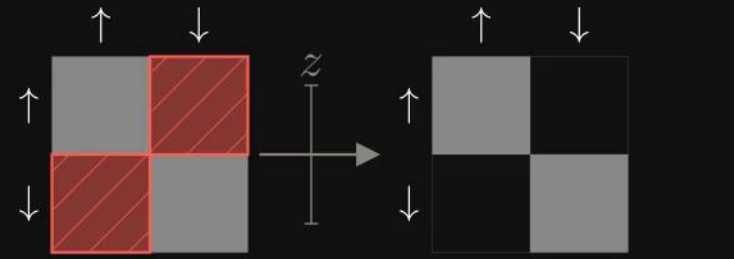
Vote: More certain | Less certain | The same



$$\Delta S_A^{(n)} = S^{(n)}(\rho_{A,Q}) - S^{(n)}(\rho_A)$$

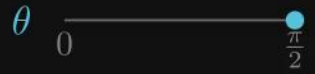


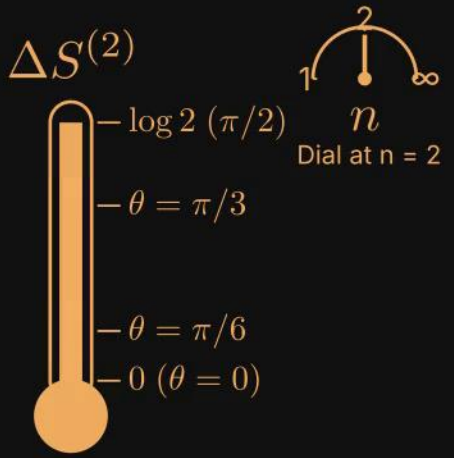
side view: x-z



secret z measurement

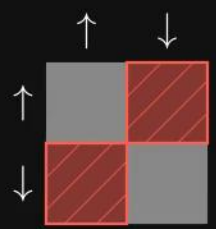
$\mathrm{diag}(\cos^2\frac{\theta}{2},\ \sin^2\frac{\theta}{2})$





$$\Delta S^{(2)} = -\log\left(\cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2}\right)$$

one spin, no partner, and the meter moves





$$\begin{pmatrix} 1/2 & c \\ c & 1/2 \end{pmatrix} \Rightarrow \log(1+4c^2)$$

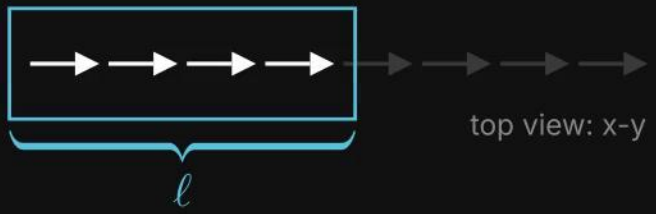
a mix of the spin along x and the fair coin; c sets the proportions



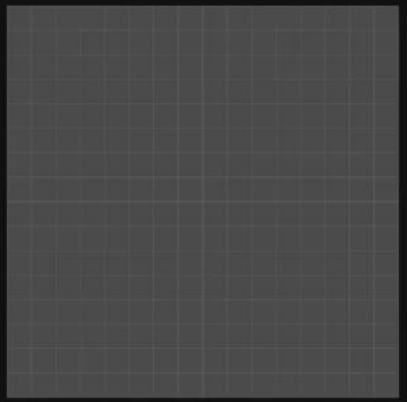
	Meter S	asymmetry
 one spin of the Bell pair	$\log 2$	0
 one spin along x	0	$\log 2$

historical name: entanglement asymmetry

The name is historical.



the window's Grid



dense: every cell $2^{-\ell}$



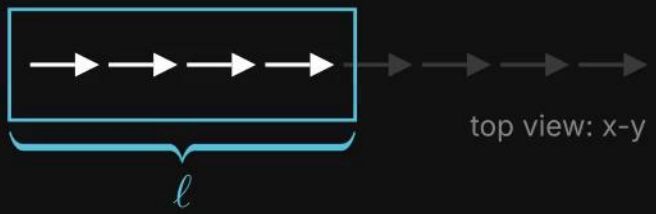
the window's Meter

? PAUSE AND PONDER

Every arrow along x and the Meter at zero. As the window grows, what does the asymmetry do?

Vote: It saturates | It grows like log ℓ | It grows like ℓ





$$P(q) = \binom{\ell}{\ell/2+q} 2^{-\ell}$$

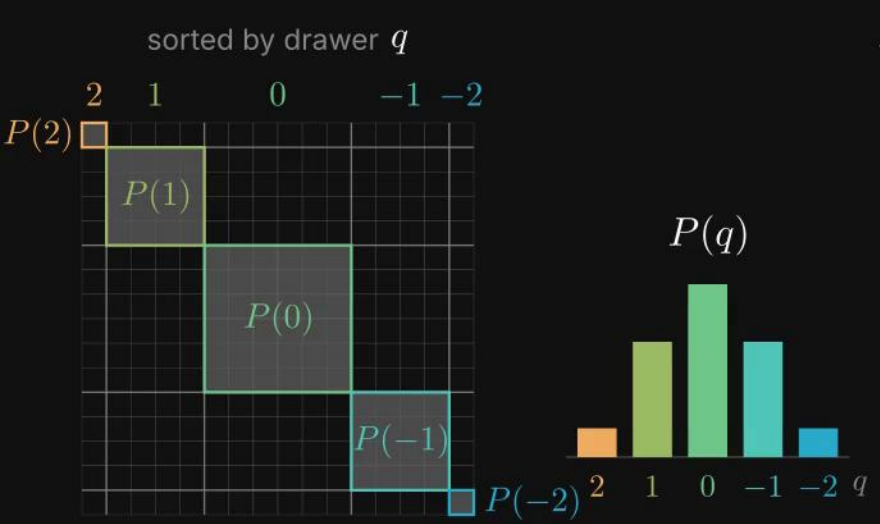
$$\Delta S^{(n)} = \frac{1}{1-n} \log \sum_q P(q)^n$$

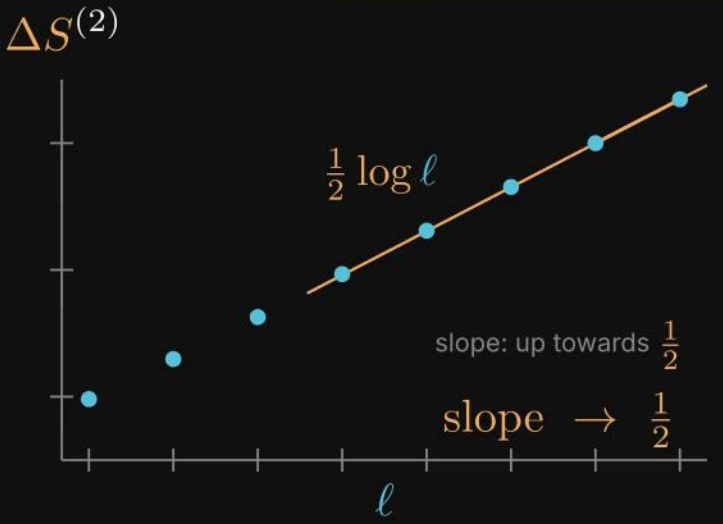
(pure)

PAUSE AND PONDER

Every arrow along x and the Meter at zero. As the window grows, what does the asymmetry do?

Vote: It saturates | It grows like log ℓ | It grows like ℓ





pure window: the eigenvalue bars of $\rho_{A,Q}$ are the charge histogram

each hand is $e^{im\alpha}$



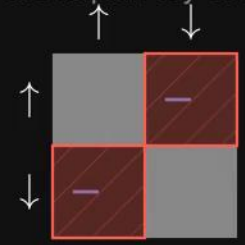
α runs once round, from $-\pi$ to π

average over a full turn: 1 if m = 0, else 0

$$\Pi_q = \int_{-\pi}^{\pi} \frac{d\alpha}{2\pi}\, e^{i\alpha(Q_A - q)}$$

the drawer projector of the sorting scene (2.3)

the napkin toy of 2.6







$$\mathrm{Tr}[\rho\,\rho(\alpha)] = \tfrac{1}{2} + 2c^2\cos\alpha$$

$$\text{circle average} = \tfrac{1}{2} + 2c^2\,\frac{\sin\alpha}{\alpha+\pi} \to \tfrac{1}{2} = \mathrm{Tr}\,{\rho_{A,Q}}^2$$

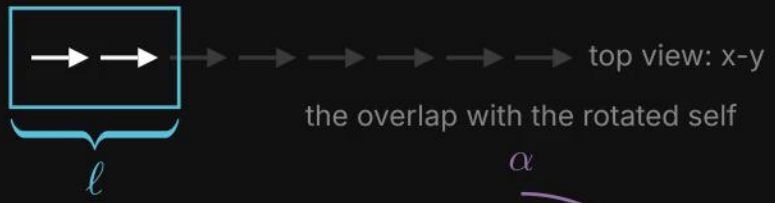
ratio, minus the log: $-\log\frac{1/2}{1/2+2c^2} = \log(1+4c^2)$

(the 2.6 formula)

the same identity holds for any table, not only the toy

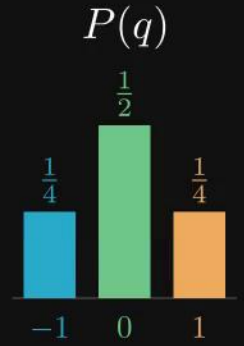
$$\frac{\mathrm{Tr}\,{\rho_{A,Q}}^2}{\mathrm{Tr}\,\rho_A^2} = \int \frac{d\alpha}{2\pi}\,Z_2(\alpha)$$

$n = 2$ is minus the log of an overlap.

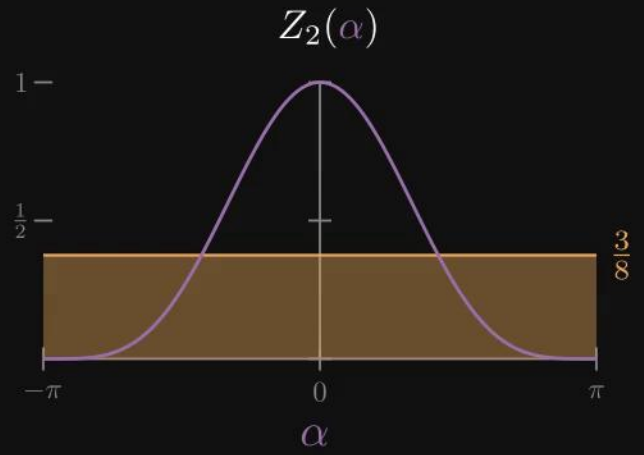


$$Z_2(\alpha) = \Big|\sum_q P(q)\, e^{i\alpha q}\Big|^2$$

(pure; mixed windows in 4.2)



charge histogram



$\alpha \in [-\pi, \pi]$

$$\sum_q P(q)\, e^{i\alpha q} = \cos^2 \tfrac{\alpha}{2}$$

$$Z_2(\alpha) = \cos^4 \tfrac{\alpha}{2}$$

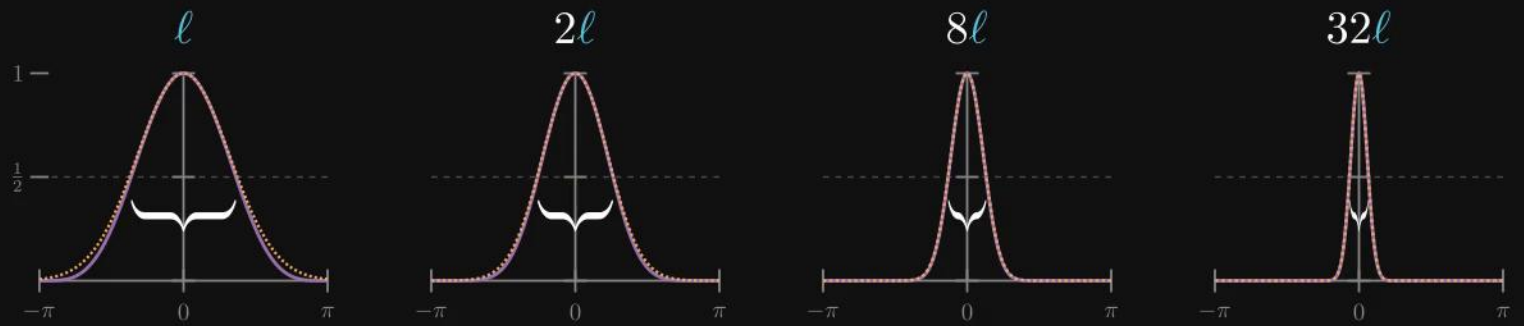
$$\int_{-\pi}^{\pi} \frac{d\alpha}{2\pi}\, Z_2(\alpha) = \tfrac{3}{8}$$

$$\Delta S_A^{(2)} = -\log \tfrac{3}{8}$$

two coins:
scene 2.7, first point

$$Z_2(\alpha) = \cos^{2\ell}(\alpha/2)$$

one factor $\cos\frac{\alpha}{2}$ per spin along x, then squared



---- $e^{-\ell\alpha^2/4}$ Gaussian

$\smile$ width $\propto 1/\sqrt{\ell}$

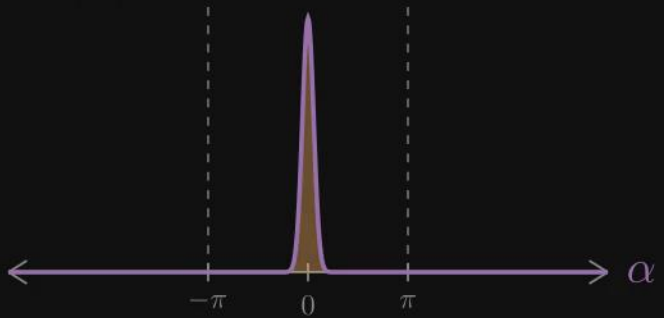
$$Z_2 \approx e^{-V\alpha^2}, \qquad V = \ell/4$$

V = the variance of the charge histogram (coins)

narrow bump in α, wide bell in q: Fourier partners

the overlap: a narrow bump in α

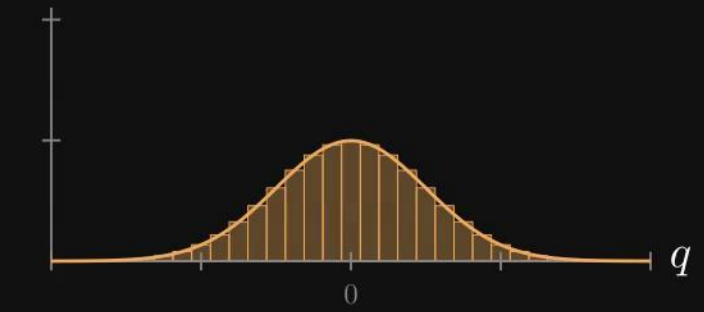
$Z_2(\alpha) \approx e^{-V\alpha^2}$



4ℓ

the Bell, variance $V = \ell/4$

$P(q)$

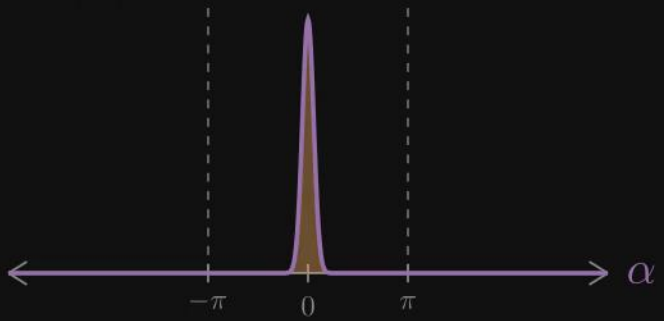


the tails we add are nothing

$$\int \frac{d\alpha}{2\pi} e^{-V\alpha^2} = \frac{1}{2\sqrt{\pi V}} \Rightarrow \Delta S_A^{(2)} = \tfrac{1}{2}\log(4\pi V) = \tfrac{1}{2}\log(\pi\ell) \text{ (coins)}$$

the overlap: a narrow bump in α

$$Z_2(\alpha) \approx e^{-V\alpha^2}$$



the tails we add are nothing

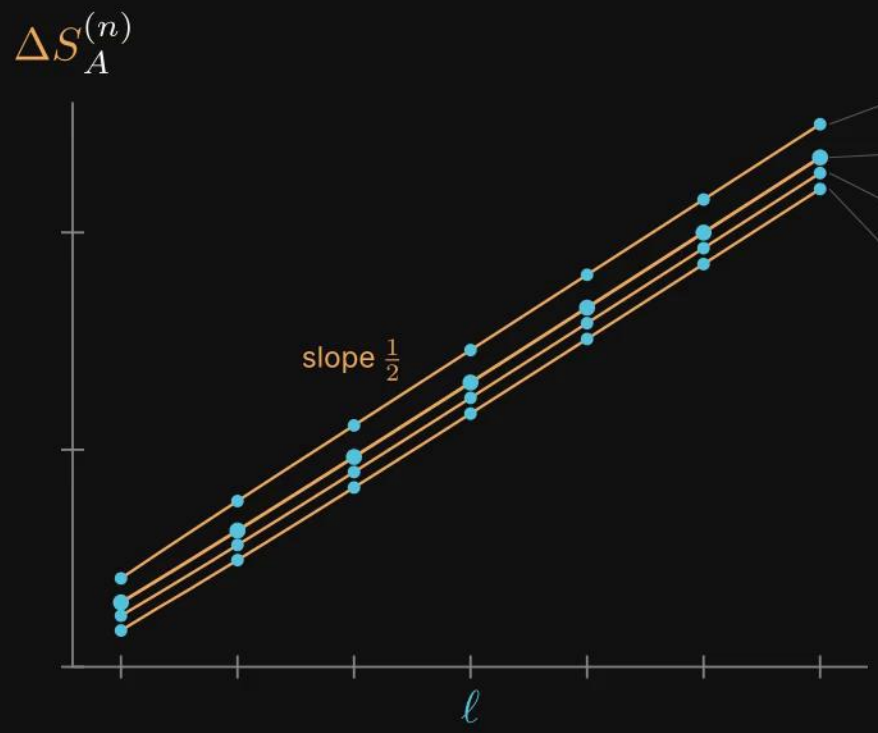
exact (binomial) against the formula

exact: $-\log\left[\binom{2\ell}{\ell}4^{-\ell}\right]$

minus the formula: $\dfrac{1}{8\ell} + \ldots$

half the log of the variance

$$\int \frac{d\alpha}{2\pi} e^{-V\alpha^2} = \frac{1}{2\sqrt{\pi V}} \Rightarrow \Delta S_A^{(2)} = \tfrac{1}{2}\log(4\pi V) = \tfrac{1}{2}\log(\pi\ell) \text{ (coins)}$$



$\frac{1}{2}\log(2\pi V) +$

$n \to 1 \quad \frac{1}{2}$

$n = 2 \quad \frac{1}{2}\log 2$

$n = 3 \quad \frac{1}{4}\log 3$ } $\frac{1}{2}\log 2 - \frac{1}{4}\log 3$

$n = 5 \quad \frac{1}{8}\log 5$



no free parameter

$$\Delta S_A^{(n)} = \frac{1}{2}\log(2\pi V) + \frac{\log n}{2(n-1)}$$

$$\Delta S_A^{(n)} = \tfrac{1}{2}\log(2\pi V) + \frac{\log n}{2(n-1)}$$

BLACK BOX · `replica-ring Gaussian integral`

- n copies of ρ_A on a closed loop, one angle per copy, one difference per junction; if every junction's bump has the same width V, the integral is Gaussian: (1/√n)(2πV)^(−(n−1)/2).
- 1/√n is the length √n of the one mode that does not move, over the product of the other mode frequencies, ∏ 2 sin(πk/n) = n (a standard identity).
- The paper (App. A) proves the bump has the same curvature V between any two copies, however far apart on the loop (V_d = V), for the states its numerics use; here it is taken on faith.

BLACK BOX · `n → 1 continuation`

- The formula is derived for whole n ≥ 2 and read at n → 1 as the limit log n / (2(n−1)) → 1/2.
- The paper confirms the n → 1 value numerically on the chain (its Fig. 2a; scene 4.4); card again in 4.3.
- The Dial of 1.8 did the same for S: the reading at n = 1 is a limit, not a separate measurement.

TAKE-HOME

$$\Delta S_A^{(n)} = S^{(n)}(\rho_{A,Q}) - S^{(n)}(\rho_A) \geq 0$$

- Sort the Grid by charge: inside a block nothing turns under a rotation about z, outside every block spins at speed q − q′; erasing the outside is what a friend's secret measurement of the charge does, and ΔS ≥ 0 is the Meter's rise.
- The asymmetry needs no entanglement: one spin along x reads log 2 with the Meter at 0; one spin of the Bell pair reads 0 with the Meter at log 2.
- The erasure is an average over all rotations (the Clock): n = 2 is minus the log of an overlap, the overlap is a Gaussian bump of width one over root ℓ, and the asymmetry is half the log of four pi V, the Bell's variance; other n differ by pure numbers.

CAVEATS

- ☐ $\Delta S \geq 0$ asserted (2.5); the proof (the erased table is an average of rotated copies, a mixture of unitaries, hence flatter eigenvalue bars) is one line of majorization, not shown.
- ☐ The circle-average identity $\mathrm{Tr}\,(\rho_{\{A,Q\}})^2 = \int d\alpha/2\pi\ \mathrm{Tr}[\rho_A\, \rho_A(\alpha)]$ was checked on the napkin toy and on a random two-spin table; the general case and every $n > 2$ are App. A of the paper (2.8).
- ☐ Pure-state shortcut throughout 2.7–2.10 ($\mathrm{Tr}\,\rho_A^2 = 1$, $Z_2 = |\Sigma_q P(q) \exp(i\alpha q)|^2$); the mixed case, where V is defined by the bump's curvature and, for the paper's states (Gaussian or gapped matrix-product windows), equals the variance up to edge terms, is 4.2; for a generic mixed table the two differ (the 4.2 toy).
- ☐ The circle integral was extended to the whole line and the bump replaced by its Gaussian (2.10); the exact value sits above the formula by $1/(8\ell)$ and smaller terms.
- ☐ Replica-loop integral, “same width at every junction” and the $n \to 1$ continuation: taken on faith (2.11); the product of mode frequencies $\prod 2\sin(\pi k/n) = n$ is a standard identity.

CAVEATS

- ☐ Rotation sign: on screen the arrows turn by −α and the Grid needles by +α (q − q′), both what exp(iαQ_A) ρ_A exp(−iαQ_A) does (1.10, 2.4).
- ☐ Odd windows have half-integer charges and integer differences; every window on screen is even (2.2).
- ☐ The small magnet is a toy: a short closed ring, exactly solved, pinned to one stripe pattern, at the home couplings Δ_z, γ; the paper's infinite-chain mark sits beside the ring's, never on its curve (scene 2.3, scene 2.5).
- ☐ “Entanglement asymmetry” is the historical name, said once (2.6); the quantum Mpemba effect was named, not explained (2.1).
- ☐ χ_zz was flashed, not defined; it arrives in 3.11 (2.12).

$$\Delta S_A^{(2)} = \tfrac{1}{2}\log(4\pi V), \quad V = \boxed{?}$$

χ_{zz}?

not defined yet: Part 3



Part 3: a magnet that prefers x over y

The meter reads half the log of the charge bell's variance. For coins it is ℓ/4. For interacting magnets whose Hamiltonian prefers x over y, what is the variance, who sets it, and is V still the variance when the piece is mixed?